1. **Part (a):** For what values of l and m does the expression $4x^4 - 12x^3 + 25x^2 - lx + m$ become a complete square, where $x \neq 0$?

2. **Part (b):** When $x^4 + kx^2 + 3$ is divided by $x - 2$, the remainder is 1. Find the value of k.

Solution for Part (a)

To make $4x^4 - 12x^3 + 25x^2 - lx + m$ a perfect square, we assume that it can be expressed as the square of a quadratic expression:

$$(2x^2 + ax + b)^2$$

Expanding $(2x^2 + ax + b)^2$ gives:

$$4x^4 + 4ax^3 + (a^2 + 4b)x^2 + 2abx + b^2$$

Now, we compare this with $4x^4 - 12x^3 + 25x^2 - lx + m$ and match coefficients for each power of x:

1. **Coefficient of x^3:** $4a = -12 \Rightarrow a = -3$.

2. **Coefficient of** x^2**:** $a^2 + 4b = 25 \Rightarrow (-3)^2 + 4b = 25 \Rightarrow 9 + 4b = 25 \Rightarrow 4b = 16 \Rightarrow b = 4.$

3. **Coefficient of** x**:** $2ab = -l \Rightarrow 2(-3)(4) = -l \Rightarrow -24 = -l \Rightarrow l = 24.$

4. **Constant term:** $b^2 = m \Rightarrow 4^2 = m \Rightarrow m = 16.$

Thus, the values of l and m are:

$$l = 24 \quad \text{and} \quad m = 16$$

Solution for Part (b)

To find the value of k such that the remainder is 1 when $x^4 + kx^2 + 3$ is divided by $x - 2$:

1. Using the **Remainder Theorem**, substitute $x = 2$ into the polynomial $f(x) = x^4 + kx^2 + 3$.
2. We have: $f(2) = 2^4 + k \cdot 2^2 + 3 = 16 + 4k +$
3. Set the remainder equal to 1: $19 + 4k = 1 \Rightarrow 4k = -18 \Rightarrow k = -$

So, the value of k is:

$$k = -\frac{9}{2}$$

Q.2

(a) Factorize the expression:

$$x^3p^2 - 8y^3p^2 - 4x^3q^2 + 32y^3q^2$$

(b) A man is currently 5 times as old as his son. Four years ago, the product of their ages was 52. Find their present ages.

Part (a)

We are asked to factorize the expression:

$$x^3p^2 - 8y^3p^2 - 4x^3q^2 + 32y^3q^2$$

To factor this expression, we can look for common factors in pairs of terms and then apply factoring techniques like grouping.

1. Group the terms:

$$(x^3p^2 - 4x^3q^2) - (8y^3p^2 - 32y^3q^2)$$

2. Factor out common factors in each group:

$$x^3(p^2 - 4q^2) - 8y^3(p^2 - 4q^2)$$

4. Now factor out $(p+2q)(p-2q)$:

$$(x^3 - 8y^3)(p+2q)(p-2q)$$

5. Recognize that $x^3 - 8y^3$ is a difference of cubes, which factors as:

$$x^3 - 8y^3 = (x-2y)(x^2 + 2xy + 4y^2)$$

6. Substitute this back:

$$(x-2y)(x^2 + 2xy + 4y^2)(p+2q)(p-2$$

Thus, the factorization is:

$$(x-2y)(x^2 + 2xy + 4y^2)(p+2q)(p-2q)$$

Part (b)

We are given that a man is currently 5 times as old as his son. Four years ago, the product of their ages was 52. We need to find their present ages.

1. **Let the son's current age be x.** Then the man's current age is $5x$.

2. **Four years ago:**
   - The son's age was $x - 4$.
   - The man's age was $5x - 4$.

3. According to the problem, four years ago, the product of their ages was 52:

$$(x - 4)(5x - 4) = 52$$

4. Expand and solve for x:

$$5x^2 - 4x - 20x + 16 = 52$$

$$5x^2 - 24x + 16 = 52$$

$$5x^2 - 24x - 36 = 0$$

5. Now, solve this quadratic equation using the quadratic formula:

$$x = \frac{-(-24) \pm \sqrt{(-24)^2 - 4 \cdot 5 \cdot (-36)}}{2 \cdot 5}$$

$$x = \frac{24 \pm \sqrt{576 + 720}}{10}$$

$$x = \frac{24 \pm \sqrt{1296}}{10}$$

$$x = \frac{24 \pm 36}{10}$$

This gives two solutions:

$$x = \frac{24 + 36}{10} = 6 \quad \text{or} \quad x = \frac{24 - 36}{10} = -1.2$$

Since age cannot be negative, $x = 6$.

7. Therefore:

    - The son's current age is 6.
    - The man's current age is $5 \times 6 = 30$.

Answer: The present ages are:

- Son: 6 years
- Man: 30 years

Q.3

(a) Solve and graph the inequality:

$$\frac{3x+4}{5} - \frac{x+1}{3} > 1 - \frac{x+5}{3}$$

(b) Find the value of $a^2 + b^2 + c^2$ given that $ab + bc + ca = 11$ and $a + b + c = 6$.

Part (a)

We are given the inequality:

$$\frac{3x+4}{5} - \frac{x+1}{3} > 1 - \frac{x+5}{3}$$

1. **Clear the Fractions**: Multiply every term by 15 (the least common multiple of 5 and 3) to eliminate the denominators:

   $$15 \cdot \frac{3x+4}{5} - 15 \cdot \frac{x+1}{3} > 15 \cdot 1 -$$

2. Simplify each term:

   $$3(3x+4) - 5(x+1) > 15 - 5(x +$$

   $$9x + 12 - 5x - 5 > 15 - 5x - 25$$

3. Combine like terms:

$$4x + 7 > -10 - 5x$$

4. Move all terms involving x to one side and constants to the other:

$$4x + 5x > -10 - 7$$

$$9x > -17$$

5. Divide by 9:

$$x > -\frac{17}{9}$$

6. **Graph the Solution**:

- On a number line, mark $x = -\frac{17}{9}$.
- Since $x > -\frac{17}{9}$, shade the region to the right of $-\frac{17}{9}$ and use an open circle at $-\frac{17}{9}$ (indicating that it's not included in the solution).

Part (b)

We need to find the value of $a^2 + b^2 + c^2$ given:

1. $ab + bc + ca = 11$
2. $a + b + c = 6$
3. **Square the Sum** $(a + b + c)^2$:

   $$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab +$$

4. Substitute the known values:

   $$6^2 = a^2 + b^2 + c^2 + 2 \cdot 11$$

   $$36 = a^2 + b^2 + c^2 + 22$$

5. Solve for $a^2 + b^2 + c^2$:

   $$a^2 + b^2 + c^2 = 36 - 22$$

   $$a^2 + b^2 + c^2 = 14$$

Answer:

$$a^2 + b^2 + c^2 = 14$$

Q4

(a) Solve $\sqrt{3x-2}-\sqrt{x}=2$.

To solve, isolate the square roots and square both sides to eliminate them, then solve the resulting equation.

(b) Simplify the expression:

$$\frac{x^4-y^4}{x^2-2xy+y^2}\times\frac{x-y}{x(x+y)}+\frac{x^2+y^2}{x}.$$

Factor and cancel common terms where possible, and then simplify each part.

Q4 (a)

Problem: Solve $\sqrt{3x-2} - \sqrt{x} = 2$.

Solution:

1. **Isolate one of the square roots:**

$$\sqrt{3x-2} = \sqrt{x} + 2$$

2. **Square both sides to eliminate the square root:**

$$3x - 2 = (\sqrt{x} + 2)^2$$

3. **Expand the right side:**

$$3x - 2 = x + 4\sqrt{x} + 4$$

4. **Move all terms involving x to one side:**

$$2x - 6 = 4\sqrt{x}$$

5. **Isolate the square root term:**

$$\frac{2x-6}{4}=\sqrt{x}$$

Simplify this:

$$\frac{x-3}{2}=\sqrt{x}$$

6. **Square both sides again to eliminate the square root:**

$$\left(\frac{x-3}{2}\right)^2=x$$

7. **Expand and simplify:**

$$\frac{(x-3)^2}{4}=x$$

Multiply both sides by 4 to clear the denominator:

$$(x-3)^2 = 4x$$

8. **Expand $(x-3)^2$:**

$$x^2 - 6x + 9 = 4x$$

9. **Bring all terms to one side to set up a quadratic equation:**

$$x^2 - 10x + 9 = 0$$

10. **Solve the quadratic equation using the quadratic formula:**

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$$

$$x = \frac{10 \pm \sqrt{100 - 36}}{2}$$

$$x = \frac{10 \pm \sqrt{64}}{2}$$

$$x = \frac{10 \pm 8}{2}$$

So, $x = \frac{10+8}{2} = 9$ or $x = \frac{10-8}{2} = 1$.

11. **Check both solutions in the original equation** to ensure they don't produce extraneous roots:

    - For $x = 9$:

      $$\sqrt{3 \cdot 9 - 2} - \sqrt{9} = \sqrt{27 - 2} - 3 =$$

      So $x = 9$ is a solution.

- For $x = 9$:

$$\sqrt{3 \cdot 9 - 2} - \sqrt{9} = \sqrt{27 - 2} - 3 =$$

So $x = 9$ is a solution.

- For $x = 1$:

$$\sqrt{3 \cdot 1 - 2} - \sqrt{1} = \sqrt{3 - 2} - 1 =$$

This does not satisfy the original equation, so $x = 1$ is an extraneous solution.

Final Answer:

$$x = 9$$

Q4 (b)

Problem: Simplify

$$\frac{x^4 - y^4}{x^2 - 2xy + y^2} \times \frac{x - y}{x(x+y)} + \frac{x^2 + y^2}{x}.$$

Let's go through it step-by-step.

Step 1: Simplify $\frac{x^4 - y^4}{x^2 - 2xy + y^2}$

1. Notice that $x^4 - y^4$ is a difference of squares:

$$x^4 - y^4 = (x^2 + y^2)(x^2 - y^2).$$

2. The term $x^2 - y^2$ is also a difference of squares:

$$x^2 - y^2 = (x - y)(x + y).$$

So,

$$x^4 - y^4 = (x^2 + y^2)(x - y)(x + y).$$

3. Now, let's rewrite the denominator $x^2 - 2xy + y^2$:

$$x^2 - 2xy + y^2 = (x - y)^2.$$

4. Substitute these factored forms back into the expression:

$$\frac{x^4 - y^4}{x^2 - 2xy + y^2} = \frac{(x^2 + y^2)(x - y)(x}{(x - y)^2}$$

5. Cancel one $(x - y)$ term from the numerator and denominator:

$$= \frac{(x^2 + y^2)(x + y)}{x - y}.$$

Now the expression becomes:

$$\frac{(x^2+y^2)(x+y)}{x-y} \times \frac{x-y}{x(x+y)} + \frac{x^2+y^2}{x}.$$

Step 2: Simplify the Multiplication

1. Notice that $(x-y)$ in the numerator and denominator cancel out:

$$= \frac{(x^2+y^2)(x+y)}{x(x+y)}.$$

2. Now, cancel $(x+y)$ from the numerator and denominator:

$$= \frac{x^2+y^2}{x}.$$

Now, the expression becomes:

$$\frac{x^2 + y^2}{x} + \frac{x^2 + y^2}{x}.$$

Step 3: Combine Like Terms

1. Since both terms are the same, we can add them:

$$= \frac{2(x^2 + y^2)}{x}.$$

Final Answer:

$$\frac{2(x^2 + y^2)}{x}.$$

Q.5 (a) Solve $(x+1)(x+2)+(x-5)(x+8)=6$ by using the quadratic formula.

(b) Product of two expressions is $x^4+3x^3-12x^2-20x+48$ and their LCM is $x^3+5x^2-2x-24$. Find their HCF.

Q5 (a)

Problem: Solve $(x+1)(x+2)+(x-5)(x+8)=6$ using the quadratic formula.

Solution:

1. **Expand both products:**

$$(x+1)(x+2) = x^2 + 2x + x + 2 = x^2$$

$$(x-5)(x+8) = x^2 + 8x - 5x - 40 =$$

2. **Substitute these expressions back into the equation:**

$$(x^2 + 3x + 2) + (x^2 + 3x - 40) = 6.$$

3. **Combine like terms:**

$$2x^2 + 6x - 38 = 6.$$

4. **Move all terms to one side of the equation:**

$$2x^2 + 6x - 38 - 6 = 0.$$

$$2x^2 + 6x - 44 = 0.$$

5. **Simplify the equation by dividing every term by 2:**

$$x^2 + 3x - 22 = 0.$$

6. **Use the quadratic formula** to solve for x:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

where $a = 1, b = 3$, and $c = -22$.

where $a = 1, b = 3$, and $c = -22$.

7. Substitute the values into the formula:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-22)}}{2 \cdot 1}.$$

$$x = \frac{-3 \pm \sqrt{9 + 88}}{2}.$$

$$x = \frac{-3 \pm \sqrt{97}}{2}.$$

Final Answer:

$$x = \frac{-3 + \sqrt{97}}{2} \quad \text{or} \quad x = \frac{-3 - \sqrt{97}}{2}.$$

Q5 (b)

Problem: The product of two expressions is

$$x^4 + 3x^3 - 12x^2 - 20x + 48$$

and their LCM is

$$x^3 + 5x^2 - 2x - 24.$$

Find their HCF.

Solution:

1. **Recall the relationship between the product, LCM, and HCF:** For two expressions A and B:

$$A \times B = \text{LCM}(A, B) \times \text{HCF}(A, B).$$

In this case:

$$x^4 + 3x^3 - 12x^2 - 20x + 48 = (x^3 + 5$$

2. **Divide the product by the LCM to find the HCF:** We need to perform polynomial division:

$$\frac{x^4 + 3x^3 - 12x^2 - 20x + 48}{x^3 + 5x^2 - 2x - 24}.$$

3. **Perform the polynomial division:**

   - **Step 1:** Divide the leading term of the numerator by the leading term of the denominator:

$$\frac{x^4}{x^3} = x.$$

- **Step 2:** Multiply x by the entire denominator:

$$x \cdot (x^3 + 5x^2 - 2x - 24) = x^4 + 5x^3 - 2x^2$$

- **Step 3:** Subtract this result from the original polynomial:

$$(x^4 + 3x^3 - 12x^2 - 20x + 48) - (x^4 + 5x^3$$

- **Step 4:** Repeat the process with the new polynomial $-2x^3 - 10x^2 + 4x + 48$:

$$\frac{-2x^3}{x^3} = -2.$$

- **Step 5:** Multiply -2 by the entire denominator:

$$-2 \cdot (x^3 + 5x^2 - 2x - 24) = -2x^3 - 10x^2$$

- **Step 6:** Subtract this result from the current polynomial:

$$(-2x^3 - 10x^2 + 4x + 48) - (-2x$$

Since the remainder is zero, the division is exact, and the HCF is the quotient we obtained.

Final Answer:

$$\text{HCF} = x - 2.$$