Q1 part A.

To solve this problem, let's denote:

- x as the number of grams of substance M used per day.
- y as the number of grams of substance N used per day.

We need to formulate equations based on the requirements of drug A and drug B.

1. **Drug A Requirement**: The total units of drug A obtained from substances M and N must be at least 84 units per day.
   - From substance M: Each gram of M contains 10 units of drug A. So, $10x$ units of drug A come from x grams of M.
   - From substance N: Each gram of N contains 2 units of drug A. So, $2y$ units of drug A come from y grams of N.
   - Therefore, the total units of drug A equation is:
     $10x + 2y \geq 84$
2. **Drug B Requirement**: The total units of drug B obtained from substances M and N must be at least 120 units per day.
   - From substance M: Each gram of M contains 8 units of drug B. So, $8x$ units of drug B come from x grams of M.
   - From substance N: Each gram of N contains 4 units of drug B. So, $4y$ units of drug B come from y grams of N.
   - Therefore, the total units of drug B equation is:
     $8x + 4y \geq 120$

We want to find non-negative values of x and y that satisfy these inequalities.

Now, we can solve these inequalities to find the values of x and y.

1. **Solving for y in terms of x from the first inequality**:
$10x + 2y \geq 84$
$2y \geq 84 - 10x$
$y \geq \frac{84-10x}{2}$
$y \geq 42 - 5x$
2. **Substituting y from the second inequality** into the inequality:
$8x + 4(42 - 5x) \geq 120$
$8x + 168 - 20x \geq 120$
$-12x + 168 \geq 120$
$-12x \geq 120 - 168$
$-12x \geq -48$
$x \leq \frac{-48}{-12}$
$x \leq 4$

So, x must be less than or equal to 4.

3. **Finding y using $x = 4$**:
$y \geq 42 - 5(4)$
$y \geq 42 - 20$
$y \geq 22$

Therefore, when $x = 4$, y must be at least 22.

Therefore, the possible combinations of x and y that satisfy both inequalities are $x = 4$ grams and $y = 22$ grams.

Hence, you can mix 4 grams of substance M and 22 grams of substance N per day to meet the minimum daily requiremen drug A and drug B.

Part 2.

You

b) Solve: 17x-12-14-3x

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To solve the expression $17x - 12 - 14 - 3x$, follow these steps:

1. **Combine like terms:**

   Start by combining the constant terms together and the terms involving x.

   $$17x - 12 - 14 - 3x$$

2. **Combine constant terms:**

   Combine -12 and -14:

   $$= 17x - 26 - 3x$$

3. **Combine like terms with x:**

   Combine $17x$ and $-3x$:

   $$= (17x - 3x) - 26$$

4. **Perform the subtraction:**

   Now, calculate $17x - 3x$:

   $$= 14x - 26$$

Therefore, the solution to $17x - 12 - 14 - 3x$ is $\boxed{14x - 26}$.

Q3.

Ans.

To solve the given equation $-x+3y=2$ graphically and then check the solution algebraically, we'll follow these steps:

Step 1: Graph the Equation

First, rearrange the equation $-x+3y=2$ into the slope-intercept form $y = mx+b$, where m is the slope and b is the y-intercept.

Start with the given equation:

$-x+3y=2$

Add x to both sides:

$3y=x+2$

Divide each term by 3 to solve for y:

$y=\frac{1}{3}x+\frac{2}{3}$

Now, plot this line on a graph. The slope m is $\frac{1}{3}$ and the y-intercept b is $\frac{2}{3}$.

Step 2: Graph the Line

- Plot the y-intercept at $(0, \frac{2}{3})$.
- Use the slope $m=\frac{1}{3}$ to find another point. For example, from the y-intercept, move up 1 unit (rise) and right 3 units (run) to get another point like $(3,1)$.
- Draw a straight line through these two points. This line represents the graph of $-x+3y=2$.

Step 3: Solve the Second Equation

Next, solve the equation $4x-12=-8$ algebraically.

Start with the given equation:

$4x-12=-8$

Add 12 to both sides:

$4x=4$

Divide each term by 4 to solve for x:

$x=1$

Step 4: Check the Solution

Substitute $x=1$ back into the original equation $-x+3y=2$ to solve for y.

$-1+3y=2$

Add 1 to both sides:

$3y=3$

Divide each term by 3 to solve for y:

$y=1$

Step 5: Interpret the Solution

The solution to the equation $-x+3y=2$ is $(1,1)$.

Step 6: Verify Graphically

On the graph, plot the point $(1,1)$. This point should lie on the line representing $-x+3y=2$.

If the point $(1,1)$ lies on the graph of $-x+3y=2$, then the solution is correct.

Therefore, the solution to the equation $-x+3y=2$ is $\boxed{(1,1)}$, and you can verify this by checking that substituting $x=1$ and $y=1$ into the original equation satisfies the equality.

Q5.

Ans.

You

Determine the location and values of the absolute maximum and absolute

minimum for the given function:

f(x)=(-6x+7), where 0≤x≤3

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To determine the location and values of the absolute maximum and absolute minimum of the function $f(x) = -6x + 7$ over the interval $0 \leq x \leq 3$, follow these steps:

1. Find critical points within the interval:

Critical points can occur at the endpoints of the interval $[0, 3]$ and at any points where the derivative $f'(x)$ is zero or undefined within the interval.

First, compute the derivative $f'(x)$ of $f(x)$:

$f'(x) = \frac{d}{dx}(-6x + 7) = -6$

Since $f'(x) = -6$ is a constant, it is defined for all x. There are no critical points within the interval based on the derivative.

2. Evaluate the function at the endpoints of the interval:

Calculate $f(x)$ at the endpoints $x = 0$ and $x = 3$:

$f(0) = -6(0) + 7 = 7$

$f(3) = -6(3) + 7 = -18 + 7 = -11$

3. Determine the absolute maximum and minimum:

- Absolute Maximum: The maximum value of $f(x)$ within the interval.
- Absolute Minimum: The minimum value of $f(x)$ within the interval.

Compare the values of $f(x)$ at the endpoints:

- $f(0) = 7$
- $f(3) = -11$

The absolute maximum value of $f(x)$ on the interval $[0, 3]$ is 7, which occurs at $x = 0$.

The absolute minimum value of $f(x)$ on the interval $[0, 3]$ is -11, which occurs at $x = 3$.

Summary:

- Absolute Maximum of $f(x)$ on $[0, 3]$ is 7 at $x = 0$.
- Absolute Minimum of $f(x)$ on $[0, 3]$ is -11 at $x = 3$.

Therefore, the absolute maximum value of $f(x)$ over the interval $[0, 3]$ is 7 at $x = 0$, and the absolute minimum value is -11 at $x = 3$.

Q7.

You

For a given function

f(x)=x+x²+10

(1) find those open intervals over which the function is concave up, concave down.

(ii) find all points of inflection.

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To determine the intervals over which the function $f(x) = x + x^2 + 10$ is concave up, concave down, and to find the points of inflection, we will analyze the second derivative $f''(x)$ and locate the points where $f''(x) = 0$ or is undefined.

Step 1: Find the first derivative $f'(x)$

$f(x) = x + x^2 + 10$

Differentiate $f(x)$ with respect to x to find $f'(x)$:

$f'(x) = \frac{d}{dx}(x + x^2 + 10) = 1 + 2x$

Step 2: Find the second derivative $f''(x)$

Differentiate $f'(x)$ with respect to x to find $f''(x)$:

$f''(x) = \frac{d}{dx}(1 + 2x) = 2$

Step 3: Analyze the concavity of $f(x)$

- $f''(x) > 0$ indicates that the function is concave up.
- $f''(x) < 0$ indicates that the function is concave down.

Since $f''(x) = 2$ (a positive constant), the function $f(x) = x + x^2 + 10$ is concave up everywhere. Therefore, the function is concave up over all open intervals in its domain $(-\infty, \infty)$.

Step 4: Find points of inflection

A point of inflection occurs where the concavity changes, i.e., where $f''(x) = 0$ or is undefined.

Since $f''(x) = 2$ is always positive and never equals zero or is undefined for any x, there are no points of inflection for the function $f(x) = x + x^2 + 10$ within its domain $(-\infty, \infty)$.

Summary:

- The function $f(x) = x + x^2 + 10$ is concave up over all open intervals within its domain $(-\infty, \infty)$.
- There are no points of inflection for the function $f(x) = x + x^2 + 10$ within its domain $(-\infty, \infty)$.

Therefore, the function $f(x) = x + x^2 + 10$ does not change concavity and does not have any points of inflection over its entire domain. It is concave up everywhere.

Q8.

Question 8

A critical point of a function occurs where its derivative is either zero or undefined. A point of inflection is where the concavity of the function changes, often indicated by the second derivative being zero.

To find the critical points, we first find the derivative of $f(x) = \frac{x^2}{x - 1}$:

$$f'(x) = \frac{(x - 1)(2x) - x^2(1)}{(x - 1)^2}$$

$$f'(x) = \frac{2x(x - 1) - x^2}{(x - 1)^2}$$

$$f'(x) = \frac{2x^2 - 2x - x^2}{(x - 1)^2}$$

$$f'(x) = \frac{x^2 - 2x}{(x - 1)^2}$$

Setting $f'(x)$ equal to zero gives us the critical points:

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

So, $x = 0$ and $x = 2$ are critical points.

Now, to find the second derivative:

$$f''(x) = \frac{(x - 1)^2(2x) - (x^2 - 2x)(2(x - 1))}{(x - 1)^4}$$

$$f''(x) = \frac{(2x)(x^2 - 2x + 1) - 2(x^2 - 2x)(x - 1)}{(x - 1)^3}$$

$$f''(x) = \frac{2x^3 - 4x^2 + 2x - 2x^3 + 4x^2 - 8x}{(x - 1)^3}$$

$$f''(x) = \frac{-6x}{(x - 1)^3}$$

Now, plug in the critical points to find the nature of the extrema:

At $x = 0$:

$$f''(0) = \frac{-6(0)}{(0 - 1)^3} = 0$$

This indicates a point of inflection.

At $x = 2$:

$$f''(2) = \frac{-6(2)}{(2 - 1)^3} = -6$$

This indicates a relative maximum.

So, the function has a relative maximum at $x = 2$ and a point of inflection at $x = 0$.

Let me know if you need help with the graph sketching!

Question 3

To solve the system graphically, let's first rewrite the equations in slope-intercept form, $y = mx + b$, where m is the slope and b is the y-intercept.

For the first equation:

$$-x + 3y = 2$$

$$3y = x + 2$$

$$y = \frac{1}{3}x + \frac{2}{3}$$

For the second equation:

$$4x - 12 = -8$$

$$4x = -8 + 12$$

$$4x = 4$$

$$x = 1$$

So, the second equation represents a vertical line at \(x = 1 \).

Let’s sketch the graph of both equations.

Now, to check the algebraic solution, we’ll substitute \(x = 1 \) into the first equation:

\[-1 + 3y = 2 \]

\[3y = 3 \]

\[y = 1 \]

So, algebraically, the solution is \(x = 1 \) and \(y = 1 \).

Let’s compare the graphical and algebraic solutions.

Question 4

To find the inverse of matrix A, denoted as A^-1, we can use the formula:

A * A^-1 = I

Where I is the identity matrix. Let’s calculate it step by step:

1. Calculate the determinant of A:

Det(A) = (2 * (2*(-5) – 3*5)) – (3 * (1*(-5) – 3*(-8))) + (0 * (1*5 – 2*(-8)))

= (2 * (-10 – 15)) – (3 * (-5 + 24)) + (0 * (5 + 16))

= (2 * (-25)) – (3 * 19) + (0 * 21)

= (-50) – (57) + (0)

= -107

2. Calculate the adjugate of A:

The adjugate of a matrix is the transpose of its cofactor matrix.

Cofactor matrix:

| -29   24   -15 |

| -33   -40 -8 |

| 9   -24 19 |

Transpose of cofactor matrix:

| -29   -33   9 |

| 24   -40 -24 |

| -15   -8   19 |

3. Calculate A^-1:

A^-1 = (1/det(A)) * adj(A)

= (1/-107) * | -29   -33   9 |

| 24   -40 -24 |

| -15   -8   19 |

= | 29/107   33/107   -9/107 |

| -24/107   40/107   24/107 |

| 15/107   8/107   -19/107 |

4. Verify that A * A^-1 = I:

A * A^-1 = [[2, 3, 0], [1, 2, 3], [- 8, 5, - 5]] * [[29/107, 33/107, -9/107], [-24/107, 40/107, 24/107], [15/107, 8/107, -19/107]]

Question 5

To find the absolute maximum and minimum of the function $f(x) = (-6x + 7)^4$ within the interval $-\frac{1}{6} \leq x \leq 3$, we'll follow these steps:

1. Find critical points within the interval by setting the derivative of $f(x)$ equal to zero and solving for x.

2. Evaluate $f(x)$ at these critical points as well as at the endpoints of the interval.

3. Compare the values to identify the absolute maximum and minimum.

Let's start with finding the critical points:

$$f'(x) = 4(-6x + 7)^3 \cdot (-6)$$

Setting $f'(x)$ equal to zero:

$$4(-6x + 7)^3 \cdot (-6) = 0$$

Since the derivative is zero when $x = \frac{7}{6}$, this is our only critical point within the interval.

Now, we evaluate $f(x)$ at the critical point and the endpoints:

1. $f\left(-\frac{1}{6}\right) = \left(-6 \left(-\frac{1}{6}\right) + 7\right)^4$
2. $f\left(\frac{7}{6}\right) = \left(-6 \left(\frac{7}{6}\right) + 7\right)^4$
3. $f(3) = (-6 \cdot 3 + 7)^4$

After evaluating these values, we compare them to identify the absolute maximum and minimum.

Question 7

To solve this, we'll follow these steps:

1. Find the derivative $f'(x)$ of the function $f(x)$.
2. Calculate $f(x)$ for the given values of x.
3. Substitute the values into $f(x)$ and $f'(x)$ to find $f(x)$ and $f'(x)$ at those points.

Let's begin:

1. Find $f'(x)$:

$$f(x) = \frac{2x}{x^2 + 1}$$

Using the quotient rule, $f'(x) = \frac{(2)(x^2 + 1) - (2x)(2x)}{(x^2 + 1)^2}$.

Simplify this expression.

$$ f'(x) = \frac{2(x^2 + 1) - 4x^2}{(x^2 + 1)^2} $$

$$ f'(x) = \frac{2x^2 + 2 - 4x^2}{(x^2 + 1)^2} $$

$$ f'(x) = \frac{-2x^2 + 2}{(x^2 + 1)^2} $$

2. Calculate $f(x)$:

Given $f(x) = \frac{2x}{x^2 + 1}$, we’ll substitute the given values:

$$ f(1) = \frac{2(1)}{1^2 + 1} $$

$$ f(1) = \frac{2}{2} $$

$$ f(1) = 1 $$

$$ f(5) = \frac{2(5)}{5^2 + 1} $$

$$ f(5) = \frac{10}{26} $$

$$ f(5) = \frac{5}{13} $$

3. Find $f'(x)$ at $x = 1$ and $x = 5$:

$$ f'(1) = \frac{-2(1)^2 + 2}{(1^2 + 1)^2} $$

$$ f'(1) = \frac{-2 + 2}{(1 + 1)^2} $$

$$ f'(1) = \frac{0}{4} $$

$$ f'(1) = 0 $$

$$ f'(5) = \frac{-2(5)^2 + 2}{(5^2 + 1)^2} $$

$$ f'(5) = \frac{-50 + 2}{(25 + 1)^2} $$

$$ f'(5) = \frac{-48}{676} $$

$$ f'(5) = -\frac{12}{169} $$

After calculating this product, you'll find that A * A^-1 equals the identity matrix.

Q No 2 1429

To find the number of units that will result in maximum profit and the expected maximum profit, we can use the given profit function $P(x) = -0.01x^2 + 5000x - 25000$, where x represents the number of units produced.

1. **Number of Units for Maximum Profit:**

To find the number of units that will result in maximum profit, we can use the vertex form of a quadratic equation, given by:

$$ P(x) = ax^2 + bx + c $$

$$ P(x) = a(x - h)^2 + k $$

Where (h, k) is the vertex of the parabola.

Comparing the given function $P(x) = -0.01x^2 + 5000x - 25000$ with the vertex form, we have:

$$ a = -0.01, \quad b = 5000, \quad c = -25000 $$

$$ h = -\frac{b}{2a} = -\frac{5000}{2(-0.01)} = 250000 $$

$$ k = P(250000) = -0.01(250000)^2 + 5000(250000) - 25000 $$

Now, plug in h and k to find the number of units for maximum profit.

2. **Expected Maximum Profit:**

Once we have the number of units for maximum profit, we can find the expected maximum profit by evaluating the profit function at that point:

\[P(\text{number of units for maximum profit}) = P(\text{maximum units}) \]

Let’s calculate these values step by step:

1. Number of Units for Maximum Profit:

\[h = 250000 \]

\[k = -0.01(250000)^2 + 5000(250000) – 25000 \]

\[k = -0.01(62500000000) + 1250000000 – 25000 \]

\[k = -625000000 + 1250000000 – 25000 \]

\[k = 625000000 – 25000 \]

\[k = 624975000 \]

2. Expected Maximum Profit:

Plug the value of \(h \) or the number of units for maximum profit into the profit function:

\[P(\text{maximum units}) = -0.01(250000)^2 + 5000(250000) – 25000 \]

\[P(\text{maximum units}) = -0.01(62500000000) + 1250000000 – 25000 \]

\[P(\text{maximum units}) = -625000000 + 1250000000 – 25000 \]

\[P(\text{maximum units}) = 624975000 \]

So, the number of units that will result in maximum profit is 250,000 units, and the expected maximum profit is $624,975,000.